Dimensional Analysis Workshop

We encounter numbers every day. Most of the time the numbers are related to measurements. These measurements involve units—such as pounds, inches, cups, meters, minutes, etc…

FROM NOW ON, YOU WILL WRITE UNITS NEXT TO ALL MEASURED VALUES.

I. Conversions allow us to change a measurement from one kind of unit to another. For example, 5 minutes can be changed to 300 seconds by using the conversion 1 min = 60 seconds. Conversions don't have to be equivalence statements; for instance, if a jogger runs 1 mile in 13 minutes, the conversion could be $\frac{1\text{ mile}}{13\text{ minutes}}$ or the reverse, $\frac{13\text{ minutes}}{1\text{ mile}}$. This is not an equivalence statement, because 13 minutes does not equal 1 mile (minutes are a measure of time, miles are a measure of distance) but it can be used as a conversion (if the jogger runs 2 miles, the time it takes should be 26 minutes).

1. **Of the following, circle the conversion factors. For the ones circled, show it written as a fraction (ratio). If it's already a fraction, show a different way it can be written as a fraction.**

12 miles

3 books per week

12 inches = 1 foot

$\frac{1\text{ m}}{1000\text{ m}}$ } typo – should be: $\frac{1\text{ km}}{1000\text{ m}}$ or $\frac{1\text{ m}}{1000\text{ mm}}$

$0.50/ounce

60 miles/hour

30 lbs

12 eggs/1 dozen

What are some visual clues for identifying a conversion factor?

– 2 different units involved (can involve more than 2)

– ratio/fraction or = sign or "per"

2. **Without solving the problem, identify the conversion factor (circle it):**

The distance from the sun to Earth is 93,000,000 miles; the speed of light is 3.00×10^8 m/s. How many minutes does it take light from the sun to reach Earth? (Note: "m" means meter, not mile; "s" means seconds; minutes will be abbreviated as "min")

3. Conversion factors involve at least two different units. What are the two units involved in the conversion factor above (in Part 2)?

m = meter (length)

s = seconds (time)

4. The number that you <u>did not</u> circle (in Part 2) is the "given information". It is going to be what you start with to solve the problem.

What is the unit in your given information? miles (fill this in the first box below, Part 5)

What is the unit you need to end up with to solve the problem? minutes (fill this in the last box below, Part 5)

5. Fill in the rest of the boxes **with units only** to propose a route for solving this problem (you may not use all the boxes). Look at your list of conversion factors to get ideas of what units will be involved.

miles → km → m → s → min

you may have written something slightly different

6. Units can be used the way numbers are. We can display any conversion factor as a fraction and perform multiplication and division with them.

a. Use your given information in the first box (with nothing in the denominator). Use numbers and units.

b. At the very end of the problem, write the units you want to end up with (leave space for the number).

c. Figure out a way to fill in the rest of the set up with conversion factors until you end up with the units that you need. Remember, units that are in the numerator and denominator can cancel out. Units in the numerators get multiplied, units in the denominators get multiplied. Use the same approach as in Jabberwocky.

$$\left(\frac{93{,}000{,}000 \text{ mi}}{}\right) \times \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right) \times \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \times \left(\frac{1 \text{ s}}{3.00 \times 10^8 \text{ m}}\right) \times \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \underline{8.3 \text{ min}}$$

II. You learned that you should use the given information to start a problem. In the previous example, the given information was not a conversion factor. In this example, the given information is also a conversion factor. As always, start with the given information.

1. The speed limit on the highway is 60. miles per hour. What is the speed limit in km per minute?

What is the other possible way the speed limit can be written (as a fraction)? Which one should you use?

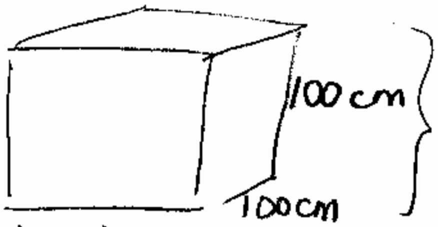
$\frac{mi}{hr}$ or $\frac{hr}{mi}$ we want mi/hr

2. Using your conversion chart, set up this problem using dimensional analysis (just as you did in section I.) This time you will begin with units in the numerator AND the denominator.

use 2 SFs due to 60. mph

$$\frac{60. \text{ mi}}{1 \text{ hr}} \times \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right) \times \left(\frac{1 \text{ hr}}{60 \text{ min}}\right) = \frac{1.61 \text{ km}}{\text{min}} \text{ or } 1.61 \text{ km/min}$$

III. Draw two cubes. On one cube show that each side is equal to 1 cm. On the other cube show each side is equal to 1 m. If the volume of a cube is the length of one side cubed (to the third power), what is the volume of each cube (in units of cm^3)?



Volume = 1 cm × 1 cm × 1 cm = $1^3 cm^3$ = 1 cm^3

100 cm × 100 cm × 100 cm = 1000000 cm^3

1. Complete the following:

1 cm = 0.01 m ⟶ 1 cm^3 = $\underline{(0.01)^3}$ m^3 (or 0.000001 m^3) or 10^{-6} m^3

2.54 cm = 1 in ⟶ $\underline{(2.54)^3}$ cm^3 = 1 in^3 (or 16.4 cm^3)

1 L = 1 dm^3 = $\underline{10^3}$ cm^3 (or 1000 cm^3)

2. Use the last conversion above to figure out the following problem. The density of ammonia gas under certain conditions is 0.625 g/L. Calculate its density in g/cm^3. Use a separate sheet and show your setup.

3. Aluminum is a lightweight metal (density=2.70 g/cm^3) used in aircraft construction, high-voltage transmission lines, and foils. What is its density in kg/m^3? Use a separate sheet and show your setup.

#2 Use the previous problem:

$1\,L = 1\,dm^3 = 10^3\,cm^3 \longrightarrow 1\,L = 1000\,cm^3$

we know
$1\,L = 1000\,mL$
so $1\,cm^3 = 1\,mL$

$\frac{0.625\,g}{L} \longrightarrow ?\ \frac{g}{cm^3}$

$$\frac{0.625\,g}{\cancel{L}} \times \left(\frac{1\,\cancel{L}}{1000\,\cancel{mL}}\right) \times \left(\frac{1\,\cancel{mL}}{1\,cm^3}\right) = \frac{0.000625\,g}{cm^3}$$

* 3 sig figs due to density

or $6.25 \times 10^{-4}\,g/cm^3$

better in sci. not

#3

$$\frac{2.70\,\cancel{g}}{\cancel{cm^3}} \times \left(\frac{1\,kg}{1000\,\cancel{g}}\right) \times \left(\frac{100^3\,\cancel{cm^3}}{1^3\,m^3}\right) = 2.70 \times 10^3\,\frac{kg}{m^3}$$

or
$2{,}7\bar{0}0\,\frac{kg}{m^3}$

or you can use

$$\left(\frac{1\,cm^3}{10^{-6}\,m^3}\right)$$